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whence $\alpha=\frac{3}{4}$, $\beta=-\frac{1}{4}$. Then $a=\frac{21}{16}$, $b=\frac{21}{16}$. The other roots are $\frac{5}{4}$ and $-\frac{7}{4}$.

Also solved by G. B. M. Zerr, and V. M. Spunar.

AVERAGE AND PROBABILITY.

192a. Proposed by A. H. HOLMES, Brunswick, Maine.

In a game of baccarat the dealer and each side of the table have two or three cards. The object is to get as near nine as possible, and tens and court cards do not count. If the first two cards dealt do not together amount to five, the player asks for another. If above five he does not. When the two cards amount to exactly five would the chances of the hand be bettered or diminished by drawing a third card, and how much?

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let A, B, C, D be the players, and, in order to avoid a multiplicity of solutions, we will assume cards for A, C, D when B has just five. Let A have an ace and a three; C, four and six; D, two and five, as follows:

A	*C*	*D*	*B*
1, 3	4, 6	2, 5	1, 4 or 2, 3 or 5, 10

In either case B betters his hand if he draws 1, 2, 3, 4, 5, 6, or 7; diminishes it if he draws 9 and leaves it the same if he draws 8, 10, or court cards. Since there are but 44 cards left in the pack, we have:

$$\text{First}\quad\left\{\begin{array}{l}\frac{2}{44}+\frac{3}{44}+\frac{3}{44}+\frac{2}{44}+\frac{3}{44}+\frac{3}{44}+\frac{4}{44}=\frac{10}{22}=\frac{5}{11}=\text{chance of bettering.}\\ \frac{4}{44}=\frac{1}{11}=\text{chance of diminishing.}\\ \frac{4}{44}+\frac{4}{44}+\frac{4}{44}+\frac{4}{44}+\frac{4}{44}=\frac{5}{11}=\text{chance of leaving same.}\end{array}\right.$$

$$\text{Second}\quad\left\{\begin{array}{l}\frac{3}{44}+\frac{2}{44}+\frac{2}{44}+\frac{3}{44}+\frac{3}{44}+\frac{3}{44}+\frac{4}{44}=\frac{5}{11}=\text{chance of bettering.}\\ \text{The chance of diminishing or leaving the same}\\ \text{is }\frac{1}{11}\text{ or }\frac{5}{11}\text{ as before.}\end{array}\right.$$

$$\text{Third}\quad\left\{\begin{array}{l}\frac{3}{44}+\frac{3}{44}+\frac{3}{44}+\frac{3}{44}+\frac{2}{44}+\frac{3}{44}+\frac{4}{44}=\frac{21}{44}=\text{chance of bettering.}\\ \text{The chance of diminishing}=\frac{4}{44}=\frac{1}{11}.\\ \frac{4}{44}+\frac{4}{44}+\frac{4}{44}+\frac{4}{44}+\frac{3}{44}=\frac{19}{44}=\text{chance of remaining the same.}\end{array}\right.$$

The solution is similar for A, C, D having other cards.

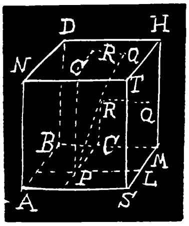
Hence we see that the drawing of a third card when the two cards count just five, is preferable. In the first and second cases, bettering to diminishing=5 : 1. In the third case, bettering to diminishing=21 : 4.

197. Proposed by HENRY HEATON, Belfield, N. D.

Solve No. 188 on the supposition that all lines having the same direction are equally distributed in space, and lines passing through the same point are distributed as the radii of a sphere drawn to points equally distributed.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

We will solve (b) first.



Let AH be the cube, P the point of ingress, Q the point of egress of the hole. Let (x, y) be the coordinates of P. Through P, pass the plane RPC parallel to the face $ABDN$. Through Q draw the line QR parallel to BM and DH, where $BMDH$ is a face of the cube. Let $LM=x$, $LP=y$, $\angle RPC=\theta$, $\angle QRP=\phi$, $a=$ side of cube. When P, Q are in the adjacent faces $ASMB$ and $BMHD$, then $PR=x\sec\theta$, $PQ=x\sec\theta\sec\phi$. The limits of θ are 0 and $\tan^{-1}(a/x)=\theta_1$; of ϕ, 0 and $\tan^{-1}(y\cos\theta/x)=\phi_1$. When P is in $ASMB$ and Q in the opposite face $NTHD$, $PR=a\sec\theta$, $PQ=a\sec\theta\sec\phi$. The limits of θ are 0 and $\tan^{-1}(x/a)=\theta_2$; of ϕ, 0 and $\tan^{-1}(y\cos\theta/a)=\phi_2$; the limits of both x and y are 0 and a.

As P may be regarded as fixed, there are four faces adjacent to $ASMB$ and one opposite. The points of intersection of PQ with the surface of a sphere, center P and radius unity, is the required distribution. An element of surface of this sphere is $\cos\phi\, d\theta\, d\phi$. Hence the average length, L, is

$$L=\frac{\int_0^a\int_0^a\left[4\int_0^{\theta_1}\int_0^{\phi_1}x\sec\theta\sec\phi\cos\phi\, d\theta\, d\phi\right.}{\int_0^a\int_0^a\left[4\int_0^{\theta_1}\int_0^{\phi_1}\cos\phi\, d\theta\, d\phi\right.}\times$$

$$\times\frac{\left.+\int_0^{\theta_2}\int_0^{\phi_2}a\sec\theta\sec\phi\cos\phi\, d\theta\, d\phi\right]dx\, dy}{\left.+\int_0^{\theta_2}\int_0^{\phi_2}\cos\phi\, d\theta\, d\phi\right]dx\, dy}=\frac{N}{D}.$$

$$D=\int_0^a\int_0^a\left[4\int_0^{\theta_1}\frac{y\cos\theta}{\sqrt{(x^2+y^2\cos^2\theta)}}d\theta+\int_0^{\theta_2}\int\frac{y\cos\theta}{\sqrt{(a^2+y^2\cos^2\theta)}}d\theta\right]dx\, dy$$

$$=\int_0^a\int_0^a\left[4\sin^{-1}\left(\frac{ay}{\sqrt{(a^2+x^2)}\sqrt{(x^2+y^2)}}\right.\right.$$

$$\left.\left.+\sin^{-1}\left(\frac{xy}{\sqrt{(a^2+x^2)}\sqrt{(a^2+y^2)}}\right)\right]dx\, dy=\int_0^a\left[4a\sin^{-1}\left(\frac{a^2}{a^2+x^2}\right)\right.$$

$$-4x\log\left[\frac{[a+\sqrt{(a^2+x^2)}]\sqrt{(a^2+x^2)}}{x[a+\sqrt{(2a^2+x^2)}]}\right]+a\sin^{-1}\left(\frac{x}{\sqrt{(2)}\sqrt{(a^2+x^2)}}\right)$$

$$-x\log\left(\frac{\sqrt{(2)}\,[x+\sqrt{(a^2+x^2)}]}{x+\sqrt{(2a^2+x^2)}}\right)\Bigg]dx$$

$$=(a^2/12)\,[10\pi+24-45\sqrt{2}+21\sqrt{3}+6\log 2-24\log(1+\sqrt{3})+27\log(1+\sqrt{2})]$$
$$=2.6653a^2.$$

$$N=4\int_0^a\int_0^a\int_0^a \frac{x}{\sqrt{(x^2+z^2)}}\tan^{-1}\left(\frac{y}{\sqrt{(x^2+z^2)}}\right)dz\,dx\,dy$$

$$+\int_0^a\int_0^z\int_0^a \frac{a}{\sqrt{(a^2+z^2)}}\tan^{-1}\left(\frac{y}{\sqrt{(a^2+z^2)}}\right)dz\,dx\,dy$$

In the first term z is written for $x\tan\theta$, in the second term z is written for $a\tan\theta$.

$$N=4\int_0^a\int_0^a \left[\frac{ax}{\sqrt{(x^2+z^2)}}\tan^{-1}\left(\frac{a}{\sqrt{(x^2+z^2)}}\right)\right.$$

$$\left.-\tfrac{1}{2}x\log\left(\frac{a^2+x^2+z^2}{x^2+z^2}\right)\right]dz\,dx+\int_0^a\int_0^z\left[\frac{a^2}{\sqrt{(a^2+z^2)}}\tan^{-1}\left(\frac{a}{\sqrt{(a^2+z^2)}}\right)\right.$$

$$\left.-\tfrac{1}{2}a\log\left(\frac{2a^2+z^2}{a^2+z^2}\right)\right]dz\,dx=\int_0^a\left[4a\sqrt{(a^2+z^2)}\tan^{-1}\left(\frac{a}{\sqrt{(a^2+z^2)}}\right)\right.$$

$$\left.-4az\tan^{-1}(a/z)+2z^2\log\left(\frac{a^2+z^2}{z\sqrt{(2a^2+z^2)}}\right)\right]dz$$

$$+\int_0^a\left[\frac{a^2z}{\sqrt{(a^2+z^2)}}\tan^{-1}\left(\frac{a}{\sqrt{(a^2+z^2)}}\right)-\tfrac{1}{2}az\log\left(\frac{2a^2+z^2}{a^2+z^2}\right)\right]dz$$

$$=\frac{a^3}{12}\left[28\sqrt{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)+8\log 2-24-7\pi-4\log 3\right]$$

$$+4a^3\int_0^{\frac{1}{4}\pi}\sec^3\psi\tan^{-1}(\cos\psi)\,d\psi. \qquad 4a^3\int_0^{\frac{1}{4}\pi}(\sec^3\psi\tan^{-1}(\cos\psi)\,d\psi$$

$$=4a^3\int_0^{\frac{1}{4}\pi}(\sec^2\psi-\tfrac{1}{3}+\tfrac{1}{5}\cos^2\psi-\tfrac{1}{7}\cos^4\psi+\tfrac{1}{9}\cos^6\psi-\tfrac{1}{11}\cos^8\psi+\tfrac{1}{13}\cos^{10}\psi-\tfrac{1}{15}\cos^{12}\psi$$

$$+\ldots)\,d\psi=4a^3(1.02697-.06868\pi)=3.2448a^3.$$

$$\therefore N=1.5390a^3,\quad L=N/D=.5774a.$$

$$\text{(a)}\quad L=\frac{2a\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\sin\theta\cos^2\phi\,d\theta\,d\phi}{\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\cos\phi\,d\theta\,d\phi}=\frac{4a}{\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\sin\theta\cos^2\phi\,d\theta\,d\phi=a.$$

MISCELLANEOUS.

174. Proposed by L. E. DICKSON, Ph. D., Associate Professor of Mathematics, The University of Chicago.

By a linear transformation with integral coefficients modulo 2, reduce $f=\Sigma x_i^2+\Sigma x_ix_j$ $(i, j=1, \ldots, 2m;\ i<j)$ to a canonical form in which the variables are separated into pairs.

Solution by the PROPOSER.

Let $x_1=y_1+s$, $x_2=y_2+s$, where $s=\sum_{i=3}^{2m} x_i$. Then (mod 2),

$$f\equiv y_1y_2+y_1^3+y_2^2+F,\quad F=\Sigma\, x_ix_j\ (i, j=3, \ldots, 2m;\ i<j).$$

In F, set $x_3=y_3+t$, $x_4=y_4+t$, where $t=\sum_{i=5}^{2m} x_i$. Then

$$F\equiv y_3y_4+G,\quad G=\Sigma\, x_i^2+\Sigma\, x_ix_j\ (i, j=5, \ldots, 2m;\ i<j),$$

modulo 2. Since G is of the form f, repetitions of the process evidently lead to the required canonical form.

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PROBLEMS FOR SOLUTION.

ALGEBRA.

317. Proposed by FRANCIS RUST, Allegheny, Pa.

Once, in classic days, Silenus lay asleep; a goat skin filled with wine near him. Dionysius passing by, profited, by siezing the skin, and drinking for two-thirds ($\frac{2}{3}$) of that time in which Silenus alone could have emptied said skin. At this point Silenus awoke, and seeing what was happening, snatched away the precious skin, and finished it.

Now, had both started together, and drank simultaneously, they would have consumed the wine skin in two hours less time. And, in this case, Dionysius' share would have been one-half as much as Silenus did secure, by waking and snatching the skin.

In what time would either one of them alone finish the goat-skin?